

CS422 Principles of Database Systems  
Multivalued Dependency

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### Motivational Example

drinker	address	beerLiked
Sue	123 Main St.	Bud
Sue	321 State St.	Pete's Ale

drinkers

- ◆ FD?? Keys??
- ◆ 3NF?? BCNF??
- ◆ Is this a good design??

### A New Form of Redundancy

drinker	address	beerLiked
Sue	123 Main St.	Bud
Sue	321 State St.	Pete's Ale



Sue	321 State St.	Bud
Sue	123 Main St.	Pete's Ale

- ◆ Any combination of address and beerLiked for Sue is a valid tuple

### Multivalued Dependency (MVD)

- ◆ A Multivalued Dependency (MVD)  $A \twoheadrightarrow B$  is an assertion that if two tuples of a relation agree on all the attributes of  $A$ , then their components in the set of attributes  $B$  may be swapped, and the result will be two tuples that are also in the relation.
- ◆ In the drinkers example:
  - $A?? B?? C=R-AB??$
  - $?? \quad ??$

### A Couple of Observations about MVD

- ◆ MVD characterizes the case where one relation tries to represent more than one *many-to-many* relationships.
- ◆ MVD vs. FD (why it's called *multivalued* dependency)

### Trivial MVD

MVD:  $A \twoheadrightarrow B$

- ◆ MVD is trivial if
  - $B \subseteq A$ , or
  - $A \cup B = R$

## MVD Rules

MVD Complementation

If  $A \twoheadrightarrow B$ , then  $A \twoheadrightarrow R - AB$

MVD Transitivity

If  $A \twoheadrightarrow B$  and  $B \twoheadrightarrow C$ , then  $A \twoheadrightarrow C - B$

MVD Augmentation

If  $A \twoheadrightarrow B$ , then  $AC \twoheadrightarrow BD$  for any  $D \subseteq C$

## MVD + FD Rules

Replication

If  $A \twoheadrightarrow B$ , then  $A \twoheadrightarrow B$

Coalescence

If  $A \twoheadrightarrow B, C$  and  $D, C \cap B = \emptyset, D \subseteq B$   
then  $A \twoheadrightarrow D$

## Proof by Chase

◆ Given a set of FDs and MVDs  $D$ , does another dependency  $d$  (FD or MVD) follow from  $D$ ?

◆ Procedure

- Start with the hypotheses of  $d$ , and treat them as "seed" tuples in a relation
- Apply the given dependencies in  $D$  repeatedly
  - If we apply an FD, we infer equality of two symbols
  - If we apply an MVD, we infer more tuples

◆ If we infer the conclusion of  $d$ , we have a proof; otherwise we have a counter-example

*From Jun Yang's lecture notes at <http://www.cs.duke.edu/~junyang>*

## Proof by Chase Example

◆ In  $R(A, B, C, D)$ , does  $A \twoheadrightarrow B$  and  $B \twoheadrightarrow C$  implies  $A \twoheadrightarrow C$ ?

Have

A	B	C	D
a	b <sub>1</sub>	c <sub>1</sub>	d <sub>1</sub>
a	b <sub>2</sub>	c <sub>2</sub>	d <sub>2</sub>

Need

A	B	C	D
a	b <sub>1</sub>	c <sub>2</sub>	d <sub>1</sub>
a	b <sub>2</sub>	c <sub>1</sub>	d <sub>2</sub>

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## Proof by Chase Example

◆ In  $R(A, B, C, D)$ , does  $A \twoheadrightarrow B$  and  $B \twoheadrightarrow C$  implies  $A \twoheadrightarrow C$ ?

Have

A	B	C	D
a	b <sub>1</sub>	c <sub>1</sub>	d <sub>1</sub>
a	b <sub>2</sub>	c <sub>2</sub>	d <sub>2</sub>

Need

A	B	C	D
a	b <sub>1</sub>	c <sub>2</sub>	d <sub>1</sub>
a	b <sub>2</sub>	c <sub>1</sub>	d <sub>2</sub>

A B

a	b <sub>2</sub>	c <sub>1</sub>	d <sub>1</sub>
a	b <sub>1</sub>	c <sub>2</sub>	d <sub>2</sub>

??

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## Union and Decomposition

◆ Union: if  $A \twoheadrightarrow B$  and  $A \twoheadrightarrow C$ , then  $A \twoheadrightarrow BC$

• Proof??

◆ Decomposition rule *no longer holds*

• Counter-example??

## Fourth Normal Form (4NF)

- ◆ A relation R is in 4NF if for every *nontrivial* MVD  $A \twoheadrightarrow B$ , A is a super key.

## Decompose into 4NF

- ◆ Find a 4NF violation  $A \twoheadrightarrow B$
- ◆ Decompose R into:
  - $R_1 = A \cup B$
  - $R_2 = (R - AB) \cup A$
- ◆ Repeat until all relations are in 4NF

## 4NF Decomposition Example

- ◆ Drinkers(name, addr, beerLiked, favBeer)
  - FD?? Key??
  - MVD??
- ◆ R(A,B,C,D) with MVD's  $A \twoheadrightarrow B$  and FD's  $A \rightarrow C$

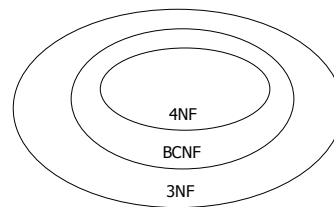
## 4NF Decomposition vs. BCNF Decomposition

- ◆ In 4NF decomposition we do not compute  $A^+$ 
  - $A^+$  does not make sense for MVD
  - $A \twoheadrightarrow (R-A)$  and  $A \twoheadrightarrow A$
- ◆ Inferring MVDs for the projections are very difficult
  - However, we can usually get by using the rules of *transitivity, complementation, and intersection.*

## Exercise: Prove the Intersection Rule

If  $A \twoheadrightarrow B$ , and  $A \twoheadrightarrow C$ , then  $A \twoheadrightarrow B \cap C$

## 4NF vs. BCNF



- ◆ Why??