CS422 Principles of Database Systems
Multivalued Dependency

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Motivational Example

drinker	address	beerLiked
Sue	123 Main St.	Bud
Sue	321 State St.	Pete's Ale

drinkers

- FD?? Keys??
- 3NF?? BCNF??
- Is this a good design??

A New Form of Redundancy

drinker	address	beerLiked
Sue	123 Main St.	Bud
Sue	321 State St.	Pete's Ale

Sue	321 State St.	Bud
Sue	123 Main St.	Pete's Ale

- Any combination of `address` and `beerLiked` for `Sue` is a valid tuple

Multivalued Dependency (MVD)

- A Multivalued Dependency (MVD) **A** **B** is an assertion that if two tuples of a relation agree on all the attributes of **A**, then their components in the set of attributes **B** may be swapped, and the result will be two tuples that are also in the relation.
- In the `drinkers` example:
  - **A**?? **B**?? **C**=**R**-**AB**??
  - ?? ??

A Couple of Observations about MVD

- MVD characterizes the case where one relation tries to represents more than one *many-to-many* relationships.
- MVD vs. FD (why it's called *multivalued* dependency)

Trivial MVD

MVD: **A** **B**

- MVD is trivial if
  - **B** ⊆ **A**, *or*
  - **A** ∪ **B** = **R**

MVD Rules

MVD Complementation
If **A** **B**, then **A** **R** - **AB**

MVD Transitivity
If **A** **B** and **B** **C**, then **A** **C**-**B**

MVD Augmentation
If **A** **B**, then **AC** **BD** for any **D** $\subseteq$ **C**

MVD + FD Rules

Replication
If **A** **B**, then **A** **B**

Coalescence
If **A** **B**, **C** **D**, **C** $\cap$ **B** = Ø , **D** $\subseteq$ **B**
then **A** **D**

Proof by Chase

- Given a set of FDs and MVDs *D*, does another dependency *d* (FD or MVD) follow from D?
- Procedure
  - Start with the hypotheses of d, and treat them as "seed" tuples in a relation
  - Apply the given dependencies in D repeatedly
    - If we apply an FD, we infer equality of two symbols
    - If we apply an MVD, we infer more tuples
- If we infer the conclusion of d, we have a proof; otherwise we have a counter-example

From Jun Yang's lecture notes at http://www.cs.duke.edu/~junyang

Proof by Chase Example

- In R(A, B, C, D), does A B and B C implies A C?

Have

A	B	C	D
a	b_1	c_1	d_1
a	b_2	c_2	d_2

Need

A	B	C	D
a	b_1	c_2	d_1
a	b_2	c_1	d_2

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Proof by Chase Example

- In R(A, B, C, D), does A B and B C implies A C?

Have

A	B	C	D
a	b_1	c_1	d_1
a	b_2	c_2	d_2

Need

A	B	C	D
a	b_1	c_2	d_1
a	b_2	c_1	d_2

A B

a	b_2	c_1	d_1
a	b_1	c_2	d_2

??

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Union and Decomposition

- Union: if **A** **B** and **A** **C**, then **A** **BC**
  - Proof??
- Decomposition rule *no longer holds*
  - Counter-example??

Fourth Normal Form (4NF)

- A relation R is in 4NF if for every *nontrivial* MVD **A** **B**, **A** is a super key.

Decompose into 4NF

- Find a 4NF violation **A** **B**
- Decompose R into:
  - $\mathbf{R_1} = \mathbf{A} \cup \mathbf{B}$
  - $\mathbf{R_2} = (\mathbf{R} - \mathbf{AB}) \cup \mathbf{A}$
- Repeat until all relations are in 4NF

4NF Decomposition Example

- Drinkers(name, addr, beerLiked, favBeer)
  - FD?? Key??
  - MVD??

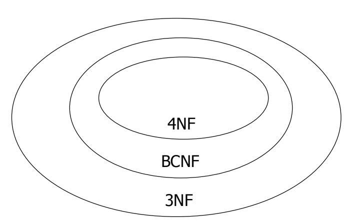
4NF Decomposition vs. BCNF Decomposition

- In 4NF decomposition we do not compute $\mathbf{A}^+$
  - $\mathbf{A}^+$ does not make sense for MVD
  - **A** (**R**-**A**) and **A** **A**
- Inferring MVDs for the projections are very difficult
  - However, we can usually get by using the rules of *transitivity*, *complementation*, and *intersection*.

Exercise: Prove the Intersection Rule

If **A** **B**, and **A** **C**, then **A** $\mathbf{B} \cap \mathbf{C}$

4NF vs. BCNF



- Why??