CS522 Advanced Database Systems

Classification: Basic Concepts and Decision Trees

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A Classification Problem

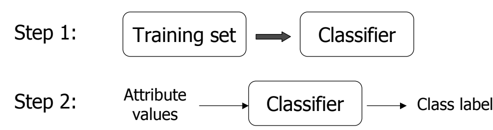
- Is a loan to a person who is 45 years old, divorced, renting an apartment, with two kids and annual income of 100K high risk or low risk?

Age	Home Owner	Marital Status	# of Kids	Annual Income	Risk
45	No	Divorced	2	100K	?

Terminology and Concepts ...

- Record (or tuple)
  - Attributes
    - E.g. age, marital status, # of kids, owns home or not, credit score ...
  - Class label
    - E.g. high risk, low risk ...
- Classification: predict the class label with given attribute values

... Terminology and Concepts



- Classifier (or model)
- Training set: records with known class labels that are used to construct (i.e. *train*) the classifier

Classification vs. Regression

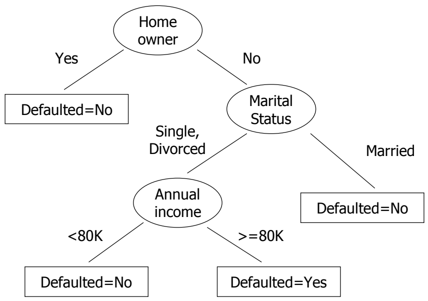
- Classification predicts categorical attribute values
- Regression predicts *continuous* numerical attribute values

SID	HW1	HW2	HW3	Final	Pass/Fail
1	40	60	70	95	Passed
2	10	15	11	65	Failed
3	30	45	40	75	Passed
4	35	50	35	?	?

A Training Set

TID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

A Decision Tree



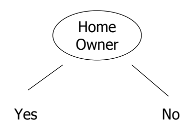
Decision Tree Induction

- Let D be the set of training record associated with current node
  - If all record in D belong to the same class C, current node is a leaf node and is labeled as C.
  - If D contains records that belong to more than one class, *select an attribute to split* D *into subsets*, and create a child node for each subset. Apply the algorithm recursively on each child node.

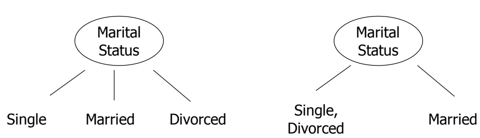
Terminating Conditions

- All records in D belong to the same class
- No more attribute to split
  - *Class label??*
- No records associated with the node
  - *Class label??*

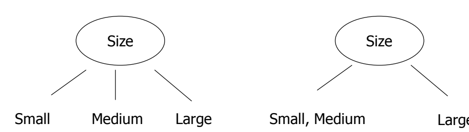
Split on Binary Attributes



Split on Nominal Attributes



Split on Ordinal Attributes



Split on Numerical Attributes …

Outlook	Humidity	Windy	Temperature	Play Golf
			85	No
			90	No
.	.	.	86	Yes
.	.	.	96	Yes
.	.	.	80	Yes
			70	No
.	.	.	65	Yes
.	.	.	95	No
.	.	.	70	Yes
			80	Yes

… Split on Numerical Attributes

- Binary split
- Pre-discretization
  - Unsupervised
    - Equi-width, equi-depth
  - Supervised
    - Entropy-based with MDL stopping Criterion

Binary Split

- Sort the values
- May split between any two adjacent values
  - May exclude splits that separate two adjacent values with the same class label
- Numerical attributes can be used to split multiple times (unlike categorical attributes)

Splitting Attribute Selection

- After a split, ideally each subset would "pure", i.e. contains only one class of records

Gender	Age	Preferred color
female	20	pink
male	20	black
female	15	pink
male	15	black

Attribute Selection Measures

- Entropy (Information Gain)
- Gini index
- Gain Ratio

Entropy

$$Entropy(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

- p_i is the fraction of records in D that belongs to class C_i
- m is the number of classes in D

Entropy Example

- Preferred color
  - 2 black and 2 pink??
  - 3 black and 1 pink??
  - 4 black??

Information Gain

- Suppose `D` is split into `v` subsets on attribute `A`

$$Gain(A) = Entropy(D) - \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Entropy(D_j)$$

Information Gain Example

- Preferred color
  - Gain(Gender)??
  - Gain(Age)??

Split Information

- Information gain favors attributes with lots of distinct values
- *Split information* can be used to "normalized" information gain

$$SplitInfo(A) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2\left(\frac{|D_j|}{|D|}\right)$$

Gain Ratio

$$GainRatio(A) = \frac{Gain(A)}{SplitInfo(A)}$$

Gini Index

$$Gini(D) = 1 - \sum_{i=1}^{m} p_i^2$$

- Used in the CART algorithm for *binary split*

Gini Index Example

- Preferred color
  - 2 black and 2 pink??
  - 3 black and 1 pink??
  - 4 black??
  - Split on gender??
  - Split on age??

Decision Tree Induction Example

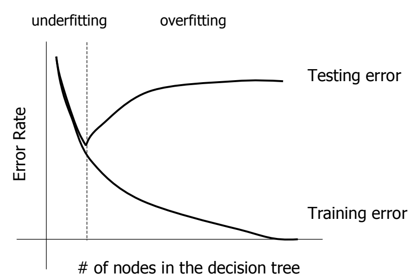
A1	A2	A3	C
Y	L	20	C1
Y	S	9	C2
N	S	11	C2
Y	M	14	C1
N	L	14	C1
Y	S	15	C1

How do we make the first split??

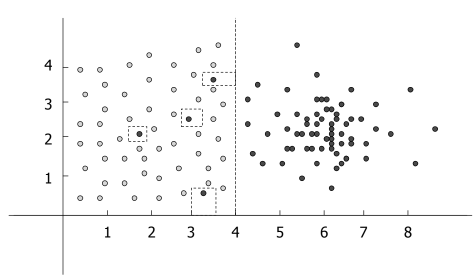
Training Error and Testing Error

- Training error
  - Misclassification of training records
- Testing (Generalization) error
  - Misclassification of testing records

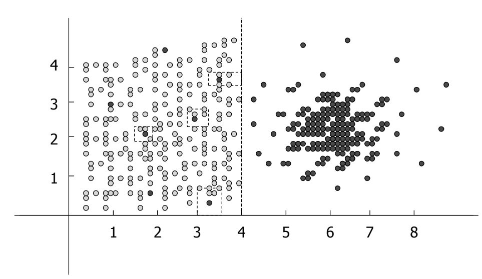
Model Overfitting and Underfitting



Overfitting Due to Outliers/Noise ...



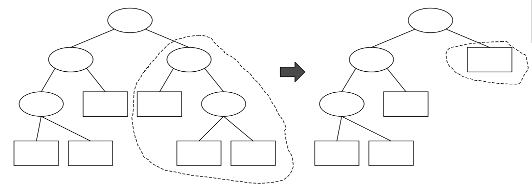
... Overfitting Due to Outliers/Noise



Occam's Razor

- A.K.A. Principle of Parsimony
- Given two models with the same generalization errors, the simpler model is preferred over the more complex model

Tree Pruning



- Replace a subtree with a leaf node
- The class label of the leaf is the majority class label of the records associated with the subtree

Prepruning

- Prune during decision tree construction
  - Number of records < threshold
  - "Purity gain" < threshold
- Performs poorly in practice

Postpruning

- Buttom-up pruning of a fully constructed tree
  - Replace a subtree with a leaf node if it reduces testing error
    - *How do we know whether it reduces testing error or not??*
  - Pruning based on Minimum Description Length (MDL)

Estimate Testing Errors ...

- Use a *pruning/validation set*
  - Usually 1/3 of the original training set
  - Less records for training

... Estimate Testing Errors

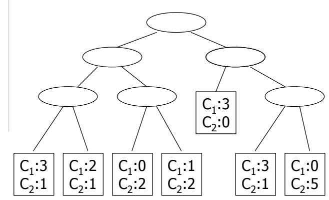
- Optimistic error estimation
  - The training set is a good representation of the overall data (optimistic!), so the training error is the testing error
- Pessimistic error estimation
  - Training error + penalty term for model complexity

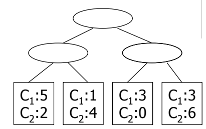
Pessimistic Error Estimation

$$e_g(T) = \frac{\sum_i [e(t_i) + \Omega(t_i)]}{\sum_i n(t_i)} = \frac{e(T) + \Omega(T)}{N}$$

- T – A decision tree
- $n(t_i)$ – # of training records at leaf node t_i
- $e(t_i)$ – # of misclassified training records at t_i
- $\Omega(t_i)$ – Penalty term associated with t_i

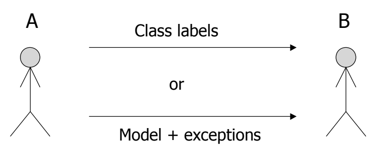
Example of Pessimistic Error Estimation





- $e_g(T)$ with $\Omega(t_i)$=0.5?? $\Omega(t_i)$=1??

Minimum Description Length (MDL)

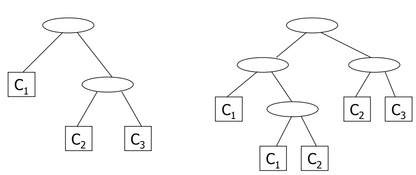


- The best model is the one that minimizes the number of bits to encode both the model and the exceptions to the model

MDL Example ...

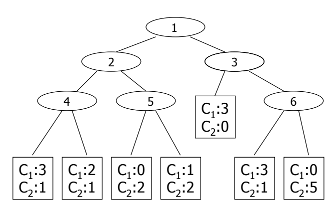
- `n` records
- `m` binary attributes
- `k` classes
- Cost(Internal Node) = $\log_2 m$
- Cost(Leaf node) = $\log_2 k$
- Cost(Error) = $\log_2 n$
- Cost = Cost(All Nodes)+Cost(All Errors)

... MDL Example



- 128 records, 8 binary attributes and 3 classes
- Left tree has 7 errors and right tree has 4 errors

Pruning with Estimated Testing Error



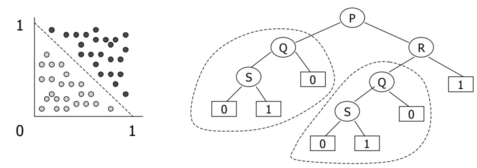
- Pessimistic error estimation with $\Omega(t_i)$=1 ??
- MDL ??

About Decision Tree Classification ...

- Inexpensive to construct
- Extremely fast at classifying unknown records
- Easy to interpret for small-sized trees
- Accuracy is comparable to other classification techniques for many simple data sets

... About Decision Tree Classification

- Data fragmentation
- Tree replication
- Finding an optimal decision tree is NP-hard
- Limitation on expressiveness



Readings

- Textbook Chapter 8.1 and 8.2