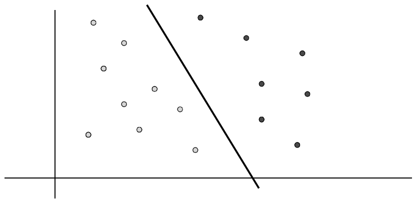
CS522 Advanced Database Systems

Classification: Introduction to Support Vector Machine

Chengyu Sun

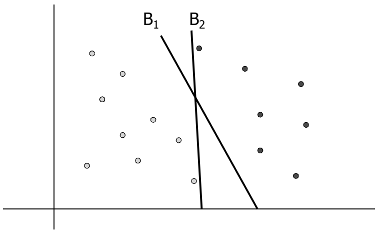
California State University, Los Angeles

Support Vector Machine (SVM)

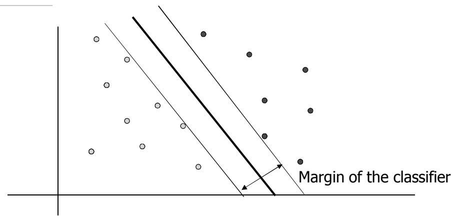


- Find a hyperplane (decision boundary) that will separate the data.

Which Boundary Is Better?



Maximum Margin Hyperplane



- Maximum margin hyperplane (MMH) minimizes the worst-case generalization error.

Linear SVM Classification

- Binary classification
- Record: $\{x_1,x_2,\ldots,x_n,y\}$
  - Attribute values: $\mathbf{X}=(x_1,x_2,\ldots,x_n)$
  - Class label: $y \in \{1, -1\}$
- Decision boundary: $\mathbf{W}\bullet\mathbf{X}+b=0$
- Classification
  - $y=1$ if $\mathbf{W}\bullet\mathbf{X}+b > 0$
  - $y=-1$ if $\mathbf{W}\bullet\mathbf{X}+b < 0$

Training SVM

	X1	X2	X3	Y
R1	0	1	0	1
R2	1	1	0	-1
R3	0	0	1	1
R4	1	0	1	-1
R5	0	0	0	-1

R1: $w_2+b \geq 1$

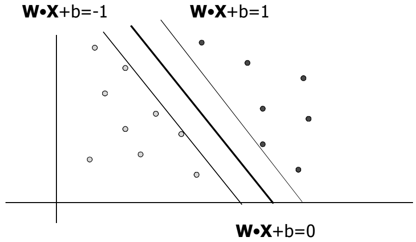
R2: $w_1+w_2+b \leq -1$

R3: $w_3+b \geq 1$

R4: $w_1+w_3+b \leq -1$

R5: $b \leq -1$

Objective Function – Maximum Margin



$$\text{Margin} = \frac{2}{\|W\|} = \frac{2}{\sqrt{w_1^2 + w_2^2 + \cdots + w_n^2}}$$

Solving Linear SVM

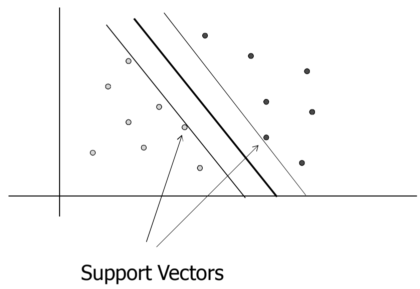
- Find **W** that satisfies the inequalities and maximize the margin 2/||**W**||
  - Constrained (convex) quadratic optimization problem
  - Solvable by numerical methods such as quadratic programming

See Chapter 5.5 of Introduction to Data Mining by Tan, Steinbach, and Kumar

Issues To Be Addressed

- Complexity when the training set is large
- Linear Non-separable case
- Non-linear decision boundary

Support Vectors

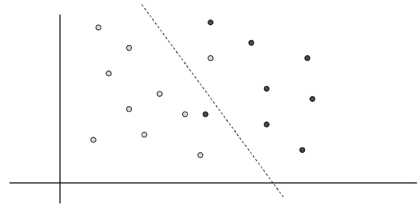


Decision Boundary of Linear SVM

$$(\sum_{i=1}^{N} \lambda_i y_i \mathbf{X_i} \bullet \mathbf{X}) + b = 0$$

- ($\mathbf{X_i}$,y_i) are training records that satisfy y_i(**W**•$\mathbf{X_i}$+b)=1, i.e. *support vectors*

Linear SVM – Non-separable Case



Introduce a Slack Variable ξ

W•X$_i$ + b ≥ 1   if y_i=1
W•X$_i$ + b ≤ -1   if y_i=-1

⬇

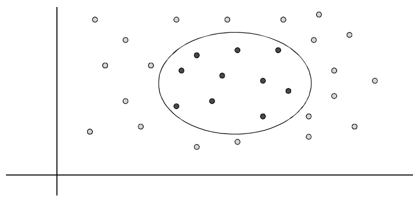
W•X$_i$ + b ≥ 1-ξ_i if y_i=1
W•X$_i$ + b ≤ -1+ξ_i if y_i=-1

Revise the Objective Function

$$f(\mathbf{W}) = \frac{\|\mathbf{W}\|^2}{2} + C(\sum_{i=1}^{N} \xi_i)^k$$

C and k are user specified parameters

Non-linear Decision Boundary



- Transform the data to another coordinate space so a linear boundary can be found

Transformation Example

Non-linear Decision Boundary in 2D space:

$$(x_1 - 1)^2 + (x_2 - 1)^2 - 1 = 0$$

$x'_1=x_1$
$x'_2=x_2$
$x'_3=x_1^2$
$x'_4=x_2^2$

Linear Decision Boundary in 4D space:

$$x'_3 - 2x'_1 + x'_4 + 2x'_2 + 1 = 0$$

Problems of Transformation

- We don't know the non-linear decision boundary (so we don't know how to do the transformation)
- Computation becomes more costly with more dimensions

Kernel Function to the Rescue

- Training records only appear in the optimization process in the form of dot product $\phi(\mathbf{X_i})\bullet\phi(\mathbf{X_j})$
  - ϕ is the transformation function
- Kernel function K($\mathbf{X_i}$,$\mathbf{X_j}$)= $\phi(\mathbf{X_i})\bullet\phi(\mathbf{X_j})$
- So we can do the computation in the original space *without even knowing what the transformation function is*

Kernel Functions

Polynomial kernel of degree h: $K(\mathbf{X}_i, \mathbf{X}_j) = (\mathbf{X}_i \bullet \mathbf{X}_j + 1)^h$

Gaussian radial basis function kernel: $K(\mathbf{X}_i, \mathbf{X}_j) = e^{-\|\mathbf{X}_i - \mathbf{X}_j\|^2 / 2\sigma^2}$

Sigmoid kernel: $K(\mathbf{X}_i, \mathbf{X}_j) = \tanh(\kappa \mathbf{X}_i \bullet \mathbf{X}_j - \delta)$

Kernel Functions and SVM Classifiers

- Use of different kernel functions result in different classifiers
- There's no golden rule to determine which kernel function is better
- The accuracy difference by using different kernel functions is usually not significant in practice

LIBSVM

- LIBSVM: a Library for Support Vector Machines by *Chih-Chung Chang* and *Chih-Jen Lin*
  - http://www.csie.ntu.edu.tw/~cjlin/libsvm/

Multiclass Classification with Binary Classifier

- Train a number of binary classifiers, each solving a binary classification problem
- Combine the results to solve the multiclass classification problem

The One-Against-Rest (1-r) Approach

- For k classes $\{c_1, c_2, \ldots, c_k\}$, train k binary classifiers M_i, each classifies $\{c_i, \text{not-}c_i\}$
  - A positive classification by M_i gives one vote to c_i
  - A negative classification by M_i gives one vote to every class other than c_i

1-r Example

- Three classes c1, c2, and c3
- Three classifiers M1, M2, and M3
- Classify record r??

Case 1:

M_1	M_2	M_3
c_1	not c_2	not c_3

Case 2:

M_1	M_2	M_3
c_1	not c_2	c_3

The One-Against-One (1-1) Approach

- For k classes $\{c_1,c_2,\ldots,c_k\}$, train k(k-1)/2 binary classifiers, each classifies $\{c_i,c_j\}$

1-1 Example

- Three classes c1, c2, and c3
- Three classifiers M1, M2, and M3
- Classify record r??

Case 1:

M_1 $\{c_1,c_2\}$	M_2 $\{c_1,c_3\}$	M_2 $\{c_2,c_3\}$
c_1	c_1	c_3

Case 2:

M_1 $\{c_1,c_2\}$	M_2 $\{c_1,c_3\}$	M_2 $\{c_2,c_3\}$
c_1	c_3	c_2

Error-Correcting Output Coding (ECOC)

- Encode each class label with a n-bit code word
- Train n binary classifiers, one for each bit
- The predicted class is the one whose codeword is the closest in Hamming distance to the classifiers' output

Error-Correcting Output Coding (ECOC) Example

Class	Codeword
c_1	1 1 1 1 1 1 1
c_2	0 0 0 0 1 1 1
c_3	0 0 1 1 0 0 1
c_4	0 1 0 1 0 1 0

- Suppose the classifiers' output: 0 1 1 1 1 1 1, what's the predicated class??

About ECOC

- If d is the minimum distance between any pair of code words, ECOC can correct up to $\lfloor (d-1)/2 \rfloor$ errors
- There are many algorithms in coding theory to generate n-bit code words with given Hamming distance
- For multiclass classification, column-wise separation is also important

Readings

- Textbook Chapter 9.3