CS522 Advanced Database Systems
Classification: Bayesian Classifiers

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Probabilistic Relationship between Attributes and Class

- Ten middle-aged, divorced, male borrowers have defaulted on their loans, but would the 11th one default as well?

Bayes Theorem

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

- Prior and posterior probabilities
  - P(A) and P(A|B)
  - P(B) and P(B|A)

Bayesian Classification

$$P(C_i \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid C_i)P(C_i)}{P(\mathbf{X})}$$

- **X** is a given record with attribute values $(x_1, x_2, \ldots, x_n)$, and C_i is a class
- $P(C_i \mid \mathbf{X})$ is the probability of **X** belonging to class C_i given **X**'s attribute values
- We predict that **X** belong to C_i if $P(C_i \mid X) > P(C_j \mid X)$ for $j \neq i$

Calculate $P(C_i|\mathbf{X})$

- $P(\mathbf{X})$ does not need to be calculated
  - *Why??*
- $P(C_i)$??
- $P(\mathbf{X}|C_i)$??

Sample Data

TID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes
11	No	Single	120K	??

Naive Bayesian Classification

- **X**=(x_1,x_2,…,x_n)
- Assume the attribute values are conditionally independent of one another (the *naive* assumption)

$$P(\mathbf{X} \mid C_i) = \prod_{i=1}^{n} P(x_i \mid C_i)$$
$$= P(x_1 \mid C_i) \times P(x_2 \mid C_i) \times \cdots \times P(x_n \mid C_i)$$

Attribute A_k is Categorical

- $P(x_k \mid C_i)$ is the fraction of number of records in C_i with value x_k for attribute A_k

Attribute A_k is Continuous-valued ...

- Assume A_k follows a Gaussian distribution with a mean μ and standard deviation σ

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2}$$

s: sample standard deviation

... Attribute A_k is Continuous-valued

$$g(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$P(x_k \mid C_i) = g(x_k, \mu_{c_i}, \sigma_{c_i})$$

Naive Bayesian Classification Example ...

- P(Default=N|HO=N,MS=S,AI=120K)
- P(Default=Y|HO=N,MS=S,AI=120K)

... Naive Bayesian Classification Example

- Annual Income, Default=No
  - μ=110, σ=54.54
  - P(AI=120K|No)=0.0072
- Annual Income, Default=Yes
  - μ=90, σ=5
  - P(AI=120K|Yes)=1.2×10^{-9}

Avoid Zero $P(x_k|C_i)$

- A zero $P(x_k|C_i)$ would make the whole $P(\mathbf{X}|C_i)$ zero
- To avoid this problem, add 1 to *each count* – assuming the training set is sufficiently large, the effect of adding one is negligible
- Example: P(Default=Y|HO=N,MS=M,AI=120K)??

About Naive Bayesian Classification

- The most accurate classification *if the conditional independence assumption holds*
- In practice, some attributes may be correlated
  - E.g. education level and annual income

Bayesian Belief Network (BBN)

- A *directed acyclic graph* (dag) encoding the dependencies among a set of variables
- A *conditional probability table* (CPT) for each node given its immediate parent nodes

A BBN Example

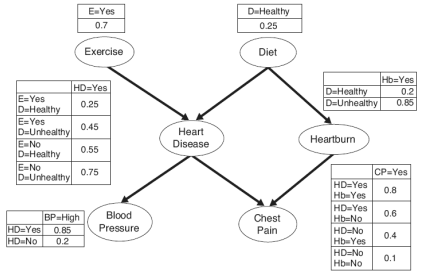


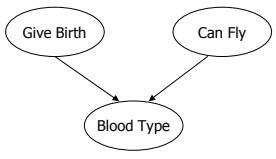
Figure 5.13. A Bayesian belief network for detecting heart disease and heartburn in patients.

Construct a BBN

- Create the structure of the network
  - From domain knowledge
  - From training data
- Calculate the CPT for each node X
  - P(X) if X does not have any parent
  - P(X|Y) if X has one parent Y
  - $P(X|Y_1,Y_2,\ldots,Y_k)$ if X has multiple parents $\{Y_1,Y_2,\ldots,Y_k\}$

BBN Construction Example

- Use the Vertebrate dataset to construct the following BBN



BBN Terminology

- If there is a directed *arc* from X to Y
  - X is a parent of Y
  - Y is a child of X
- If there is a directed *path* from X to Y
  - X is an ancestor of Y
  - Y is a descendent of X

Conditional Independence in BBN

- A node in a Bayesian network is conditionally independent of its non-descendants if its parents are known

$$P(x_1,...,x_n) = \prod_{i=1}^{n} P(x_i \mid Parents(A_i))$$

Bayesian Classification Examples

- Output node – Heart Disease
- Testing data
  - ()
  - (BP=high)
  - (BP=high,D=Healthy,E=Yes)

Bayesian Classification Examples – 1

$$P(HD=Yes) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(HD=Yes \mid E=a_i, D=b_j)P(E=a_i, D=b_j)$$
$$= \sum_{i=1}^{n}\sum_{j=1}^{m} P(HD=Yes \mid E=a_i, D=b_j)P(E=a_i)P(D=b_j)$$
$$= 0.49$$

Bayesian Classification Examples – 2

$$P(HD=Yes \mid BP=High)$$
$$= \frac{P(BP=High \mid HD=Yes)P(HD=Yes)}{P(BP=High)}$$
$$= \frac{P(BP=High \mid HD=Yes)P(HD=Yes)}{\sum_{i=1}^{n} P(BP=High \mid HD=a_i)P(HD=a_i)}$$
$$= 0.80$$

Bayesian Classification Examples – 3

$$P(HD=Yes \mid BP=High, D=Healthy, E=Yes)$$
$$= \frac{P(BP=High \mid HD=Yes, D=Healthy, E=Yes)P(HD=Yes \mid D=Healthy, E=Yes)}{P(BP=High \mid D=Healthy, E=Yes)}$$
$$= \frac{P(BP=High \mid HD=Yes)P(HD=Yes \mid D=Healthy, E=Yes)}{\sum_{i=1}^{n} P(BP=High \mid HD=a_i)P(HD=a_i \mid D=Healthy, E=Yes)}$$
$$= 0.59$$

About BBN

- Does not assume attribute independence
- Provides a way to encode domain knowledge
  - Robust to model overfitting
- Any node can be used an output node

Bayes Error Rate

- If the relationship between attributes and class is probabilistic, it is impossible to be 100% correct.
- Bayes Error Rate – minimum achievable error rate for a given classifier

Bayes Error Rate Example ...

- Identify alligators and crocodiles based on their lengths X
- P(X|Crocodile) is Gaussian with μ=15 and σ=2
- P(X|Alligator) is Gaussian with μ=12 and σ=2

...Bayes Error Rate Example...

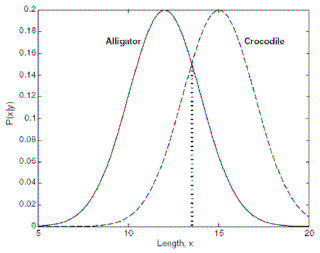


Figure 5.11. Comparing the likelihood functions of a crocodile and an alligator.

... Bayes Error Rate Example

$$Error = \int_0^{\hat{x}} P(X \mid Crocodile)dX + \int_{\hat{x}}^{\infty} P(X \mid Alligator)dX$$

$$\Downarrow$$

$$P(X = \hat{x} \mid Crocodile) = P(X = \hat{x} \mid Alligator)$$

$$\Downarrow$$

$$\hat{x} = 13.5$$

Readings

- Textbook 8.3 and 9.1.1