CS522 Advanced Database Systems

Data Cube Computation

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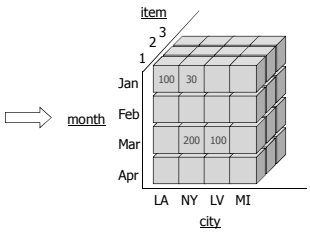
The Data

- Fact (*Measure*)
  - Sales
- Dimensions
  - Month: `1, 2, 3, 4`
  - City: `LA, NY, LV, MI`
  - Item: `1, 2, 3`

item	month	city	sales
1	Jan	LA	100
2	Feb	LA	50
1	Jan	NY	30
1	Mar	NY	200
1	Mar	LV	100
3	Apr	MI	150

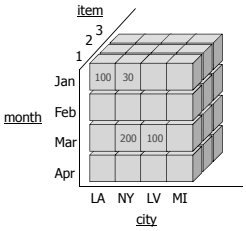
The Multidimensional Model

item	month	city	sales
1	Jan	LA	100
2	Feb	LA	50
1	Jan	NY	30
1	Mar	NY	200
1	Mar	LV	100
3	Apr	MI	150



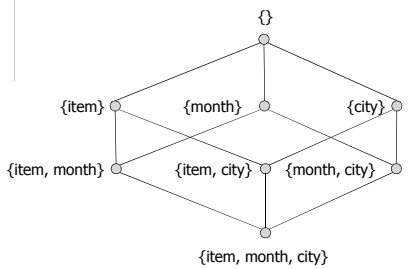
A Cuboid

- 3-D cuboid {item, month, city}



Data Cube

- A lattice of cubiods



0-D (apex) cuboid

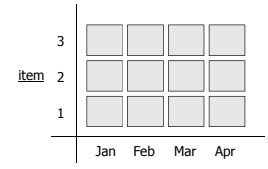
1-D cuboid

2-D cuboid

3-D (base) cuboid

More Cuboids

{item, month}



{month} ??

{} ??

About the Data Cube

- # of cuboids??
- # of cells in each cuboid??
- How do a few records turn into so much data??

Observations and Solutions

- Observations
  - Curse of Dimensionality
  - Sparsity
  - *Closed coverage*
- Solutions
  - Partial computation of data cube
    - Iceberg Cube
    - Shell Cube
  - Cube compression
    - Closed Cube

Cell

- A cell in a n-dimensional cube: $(a_1, a_2, \ldots, a_n, measure)$
- a_i is either a value or *
- A cell is a m-dimensional cell if exactly m values in $\{a_1, a_2, \ldots, a_n\}$ are not *
- Base cell: $m=n$
- Aggregate cell: $m<n$

Cell Examples

- C1: (*,*,LA,150)
- C2: (2,*,LA,50)
- C3: (1,Jan,LA,100)
- C4: (1,*,NY,230)
- C5: (*,*,NY,230)

Ancestor and Descendent Cells

- An i-D cell $a=(a_1, a_2, \ldots, a_n, measure_a)$ is an ancestor of a j-D cell $b=(b_1, b_2, \ldots, b_n, measure_b)$ iff
  - $i<j$, and
  - For $1 \le m \le n$, $a_m=b_m$ whenever $a_m \neq *$
- a is a parent of b (and b a child of a)
  - a is an ancestor of b, and
  - $j=i+1$

Ancestor and Descendent Examples

- C1: (*,*,LA,150)
- C2: (2,*,*,LA,50)
- C3: (1,Jan,LA,100)
- C4: (1,*,NY,230)
- C5: (*,*,NY,230)

Closed Cell

- A cell c is a closed cell if there is no descendent of c that has the same measure as c

Closed Cell Examples

- Which of the following are *closed cells*??
  - C1: (*,*,LA,150)
  - C2: (2,*,*,LA,50)
  - C3: (1,Jan,LA,100)
  - C4: (1,*,NY,230)
  - C5: (*,*,NY,230)

Closed Cube

- A closed cube is a data cube consisting of only *closed cells*

What's the closed cube of the following data??

item	month	city	sales
1	Jan	LA	100
2	Feb	LA	50

Query a Closed Cube

- (1,Jan,LA,??)
- (1,*,LA,??)
- (1,*,NY,??)
- (2,*,*,??)
- (*,*,*,??)

Full Cube Computation Example – Dimensions

item (a)	
a1	1
a2	2
a3	3

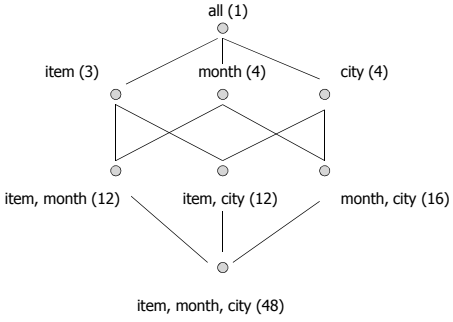
month (b)	
b1	Jan
b2	Feb
b3	Mar
b4	Apr

city (c)	
c1	LA
c2	NY
c3	LV
c4	MI

Cell examples:

$(a_1,b_1,c_1,100)$ $(a_2,*,c_3,??)$ $(*,b_2,*,??)$

Full Cube Computation Example – Data Cube



Full Cube Computation

- Approach 1: one cuboid (i.e. group-by) at a time
  - 2^n scans
- Approach 2: single scan??

Naïve Single Scan …

- Create all cube cells in memory and initialize them to 0
- Read in each record and update corresponding cells

… Naïve Single Scan

- For example, after reading (a1,b1,c1,100), the following cells will be updated:
  - (a1,b1,*), (a1,*,c1), (*,b1,c1)
  - (a1,*,*), (*,b1,*), (*,*,c1)
  - (*,*,*)

Problem with naïve single scan??

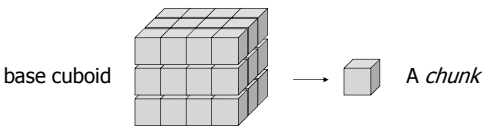
Reduce Memory Requirement - Order Matters

- Cells need to be kept in memory
  - Read unsorted: 52
  - Read sorted: ??

a	b	c	sales
a1	b1	c1	100
a1	b1	c2	30
a1	b3	c2	200
a1	b3	c3	100
a2	b2	c1	50
a3	b4	c4	150

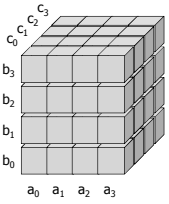
Multiway Array Aggregation

- Use a multidimensional array store the base cuboid
- Partition the array into chunks such that each chunk can fit into the memory
- Read in each chunk in certain order to compute the aggregates



MAA Example – Data

- Three dimensions
  - A: cardinality=40, partitions=4
  - B: cardinality=400, partitions=4
  - C: cardinality=4000, partitions=4



MAA Example – 3D to 2D

- To compute all the 2D cells, which of the ordering of the chunks is the best?
  - Sort by `a, b, c`
  - Sort by `b, a, c`
  - Sort by `c, b, a`

Iceberg Cubes

- Data cubes that contain only cells with aggregates greater than a minimum threshold (minimum threshold support, or *minimum support*)

The Apriori Property

- If a cell does not satisfy minimum support, then no descendant of the cell can satisfy the minimum support
- Antimonotonic aggregation functions
  - E.g. `count`, `sum`
- *Non-antimonotonic* aggregation functions
  - E.g. `avg`

BUC

- Bottom-Up Construction
- An algorithm to compute iceberg cubes with antimonotonic measures
- It's actually top-down in our view of the lattice of cuboids

BUC Example

A	B	C	Sum
a_1	b_1	c_1	5
a_2	b_1	c_2	10
a_1	b_2	c_1	3
a_2	b_1	c_1	6
a_1	b_2	c_2	4
a_2	b_2	c_3	4

- Compute an iceberg cube with `sum > 5`

BUC Outline …

- Aggregate all the input records

A	B	C	Sum
a_1	b_1	c_1	5
a_2	b_1	c_2	10
a_1	b_2	c_1	3
a_2	b_1	c_1	6
a_1	b_2	c_2	4
a_2	b_2	c_3	4

⇒ (*,*,*,32)

… BUC Outline …

- Partition the input records on the distinct values of the *next* dimension

A	B	C	Sum
a_1	b_1	c_1	5
a_2	b_1	c_2	10
a_1	b_2	c_1	3
a_2	b_1	c_1	6
a_1	b_2	c_2	4
a_2	b_2	c_3	4

⇒

a_1	b_1	c_1	5
a_1	b_2	c_2	4
a_1	b_2	c_1	3
a_2	b_1	c_1	6
a_2	b_1	c_2	10
a_2	b_2	c_3	4

… BUC Outline …

- If a partition satisfy the iceberg condition, recursively call BUC using this partition as input

a_1	b_1	c_1	5
a_1	b_2	c_2	4
a_1	b_2	c_1	3

⇒ $(a_1,*,*,12)$

… BUC Outline

- If a partition satisfy the iceberg condition, recursively call BUC using this partition as input

a_1	b_1	c_1	5
a_1	b_2	c_2	4
a_1	b_2	c_1	3

⇒

a_1	b_1	c_1	5
a_1	b_2	c_2	4
a_1	b_2	c_1	3

BUC (Bottom-Up Construction)

```
BUC( input, dim )
   aggregate(input) // place result in outputRec
   if( input.count() == 1 ) the
      WriteAncestors(input[0],dim); return;
   endif
   write outputRec
   for( d=dim ; d < numDims ; ++d )
      C = cardinality[d]
      Partition(input,d,C,dataCount[d])
      k=0
      for( i=0 ; i < C ;  ++i )
         c = dataCount[d][i]
         if c >= min_sup
            outputRec.dim[d] = input[k].dim[d]
            BUC(input[k…k+c],d+1)
         endif
         k += c
      endfor
      outputRec.dim[d] = all
   endfor
```

BUC Example

A	B	C	Sum
a_1	b_1	c_1	5
a_2	b_1	c_2	10
a_1	b_2	c_1	3
a_2	b_1	c_1	6
a_1	b_2	c_2	4
a_2	b_2	c_3	4

- Construct an Iceberg cube with `sum>5`

- Aggregates
  - (*,*,*,32)
    - $(a_1,*,*,12)$
      - ~~$(a_1,b_1,*,5)$~~
      - $(a_1,b_2,*,7)$
        - ~~$(a_1,b_2,c_1,3)$~~
        - ~~$(a_1,b_2,c_2,4)$~~
    - $(a_2,*,*,20)$
      - …
    - $(*,b_1,*21)$
      - …
    - $(*,*,c_1,14)$
      - …

A Few Optimizations in BUC

- Apriori pruning
- Dimension ordering
- Single record partition

Problems of Iceberg Cubes

- May still be too large
- Minimum support is hard to determine
- Incremental updates require re-computation of the whole cube

Cube Shells

- Observation: most OLAP operations are performed on a small number of dimensions at a time
- A cube shell of a data cube consists of the cuboids up to a certain dimension
  - E.g. all cuboids with 3 dimensions or less in a 60-dimension data cube

Problems with Cube Shells

- They may still be too large
  - E.g. how many cuboids in a 3-D shell of a 60-D data cube??
- They can't be used to answer queries like `(location,product_type,supplier,2004,?)`

Shell Fragments

- Compute only parts of a cube shell – shell fragments
- Answer queries using pre-computed *or* dynamically computed data

Shell Fragment Example

tid	a	b	c	d	e
1	a_1	b_1	c_1	d_1	e_1
2	a_1	b_2	c_1	d_2	e_1
3	a_1	b_2	c_1	d_1	e_2
4	a_2	b_1	c_1	d_1	e_2
5	a_2	b_1	c_1	d_1	e_3

Shell Fragments Construction (1)

- Partition the dimension into *non-overlapping* groups – fragments

  (a,b,c,d,e) ➔ (a,b,c) and (d,e)

Shell Fragments Construction (2)

- Scan the base cuboid and construct an inverted index for each attribute

Attribute value	TID list	List size
a_1	{1,2,3}	3
a_2	{4,5}	2
b_1	{1,4,5}	3
b_2	{2,3}	2
c_1	{1,2,3,4,5}	5
d_1	{1,3,4,5}	4
d_2	{2}	1
e_1	{1,2}	2
e_2	{3,4}	2
e_3	{5}	1

Shell Fragments Construction (3) …

- Compute the full *local* data cube (except the local apex cuboid) for each fragment
  - Vs. Cube shell??
- Record an inverted index for each cell in the cuboids

(a,b,c) ➔ a, b, c, ab, ac, bc, abc
(d,e) ➔ d, e, de

… Shell Fragment Construction (3) …

ab cuboid

Cell	Intersection	TID List	List Size
(a_1,b_1)	{1,2,3} ∩ {1,4,5}	{1}	1
(a_1,b_2)	{1,2,3} ∩ {2,3}	{2,3}	2
(a_2,b_1)	{4,5} ∩ {1,4,5}	{4,5}	2
(a_2,b_2)	{4,5} ∩ {2,3}	{}	0

- Inverted indexes are built as the cell aggregates are computed
- Apriori property can be used to prune some computation

… Shell Fragment Construction (3)

- Using an *ID_measure* array instead of the original database table

TID	Item_count	sum
1	5	70
2	3	10
3	8	20
4	5	40
5	2	30

Query Cube Fragments – Point Query

- Point query: all dimensions are instantiated with either a value or *
- Examples:
  - $(a_1,b_2,c_1,d_2,e_1,??)$
  - $(a_1,b_2,c_1,d_2,*,??)$
  - $(*,b_2,c_1,d_2,*,??)$

Answering Point Queries

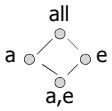
$(a_1, b_2, c_1, d_2, e_1)$ → {2,3} ∩ {2} → {2}

$(*, b_2, c_1, d_2, *)$ → ??

Query Cube Fragments – Subcube Query

- Subcube query: at least one of the dimensions is *inquired* (i.e. a group-by attribute)
- Example:

($\mathbf{a}$, b_2, c_1, *, $\mathbf{e}$, ??) ⟶ 2-D data cube on **a** and **e**



Answering Subcube Queries

($\mathbf{a}$, b_2, c_1, *, $\mathbf{e}$, ??)

a_1: {1,2,3} ∩ {2,3} ∩ e_1: {1,2}
a_2: {4,5} e_2: {3,4}
e_3: {5}

Base cuboid of **ae**

(a_1,e_1) (a_1,e_2) (a_1,e_3) (a_2,e_1) (a_2,e_2) (a_2,e_3)

Full cube computation

Data cube on **a** and **e**

OLAP Storage Types

- Relational OLAP (ROLAP)
- Multidimensional OLAP (MOLAP)
- Hybrid OLAP (HOLAP)

A ROLAP Data Store

- Summary fact tables

RID	Item	◦◦◦	Day	Month	Quarter	Year	Sales
1001	TV		15	10	Q4	2003	250
1002	TV		23	10	Q4	2003	175
◦◦◦				◦◦◦		◦◦◦	
5001	TV		**all**	10	Q4	2003	45,786

Summary

- Data cube
  - Cuboid
- Closed cube
- Full cube computation
  - Multiway Array Aggregation
- Iceberg cube
  - BUC
- Cube shell fragments
  - Construction
  - Query
- OLAP storage types

Readings

- Textbook Chapter 4.1 except 4.1.4