CS522 Advanced Database Systems
Clustering: Hierarchical Methods

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Hierarchical Clustering

- Agglomerative
  - Start with each object as a cluster
  - Recursively pick two clusters to merge
- Divisive
  - Start with all objects as a single cluster
  - Recursively pick one cluster to split

Agglomerative Hierarchical Clustering

1. Compute a *distance matrix*
2. Merge the two *closest* clusters
3. Update the distance matrix
4. Repeat Step 2 until only one cluster remains

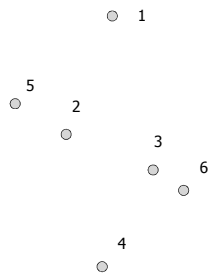
Distance Between Clusters …

- Min distance
  - Distance between two closest objects
  - Min < threshold: Single-link Clustering
- Max distance
  - Distance between two farthest objects
  - Max < threshold: Complete-link Clustering
- Average distance
  - Average of all pairs of objects from the two clusters

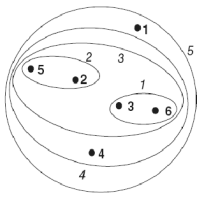
… Distance Between Clusters

- Mean distance
- Increased SSE (Ward's Method)

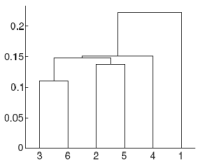
Min Distance Clustering Example ...



... Min Distance Clustering Example

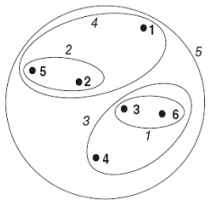


(a) Single link clustering.

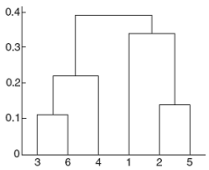


(b) Single link dendrogram.

Max Distance Clustering Example

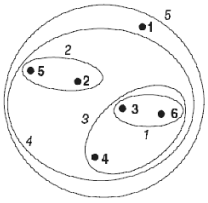


(a) Complete link clustering.

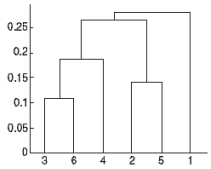


(b) Complete link dendrogram.

Average Distance Clustering Example

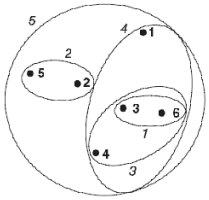


(a) Group average clustering.

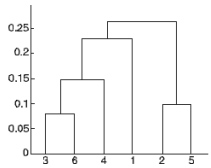


(b) Group average dendrogram.

Ward's Clustering Example



(a) Ward's clustering.



(b) Ward's dendrogram.

About Hierarchical Clustering

- Produces a hierarchy of clusters
- Lack of a global objective function
- Merging decisions are final
- *Expensive*
- Often used with other clustering algorithms

BIRCH

- Balanced Iterative Reducing and Clustering using Hierarchies
  - Designed for clustering large amount of numerical data

Clustering Feature (CF)

- A cluster can be represented by a clustering feature CF=<N, **LS**, SS>

  N: number of objects

  LS (Linear Sum): $\mathbf{LS} = \sum_{i=1}^{N} \mathbf{x}_i$

  SS (Square Sum): $SS = \sum_{i=1}^{N} \mathbf{x}_i^2 = \sum_{i=1}^{N} \mathbf{x}_i \bullet \mathbf{x}_i$

CF Example

- A cluster with two points (1,2) and (3,4)
  - N: 2
  - **LS**: (1+3,2+4) = (4,6)
  - SS: $1^2+2^2+3^2+4^2 = 30$

Incremental Update of CF

- Cluster {(1,2), (3,4)}
  - Add a point (5,6)
  - Merge with cluster {(2,3),(4,5)}

Using CF

- Centroid??
- Centroid distance??

Cluster-to-Cluster Distances

- Cluster-to-cluster distances that can be calculated using CF
  - D_0: centroid Euclidean distance
  - D_1: centroid Manhattan distance
  - D_2: average inter-cluster distance
  - D_3: average intra-cluster distance
  - D_4: variance increase distance

Cluster Diameter

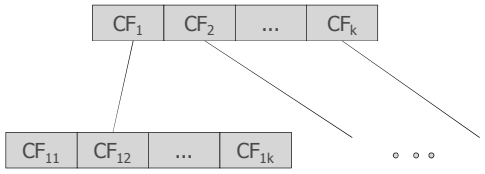
$$D = \sqrt{\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}(\mathbf{x}_i - \mathbf{x}_j)^2}{N(N-1)}} = \sqrt{\frac{2N \times SS - 2\mathbf{LS}^2}{N(N-1)}}$$

About Clustering Feature

- Space efficiency
- Computation efficiency

CF Tree

- Hierarchical clustering through tree construction (as oppose to agglomeration/division)



CF Tree Input

- Dataset
- Threshold Condition
  - Diameter `D` of a leaf node cluster < `d`

CF Tree Insertion

- Insert an object into its closest cluster in a leaf node
  - The object is added to the cluster if the resulting cluster does not violate the threshold condition
  - Otherwise the object is added as a new cluster by itself
- When a node is full, split it and rebalance the tree (similar to B+ Tree Insertion)

CF Tree Howto's

- Find closest cluster
  - *Object-to-cluster distance*
- Insert object into a cluster
  - *Update CF*
  - Check threshold condition
    - *Calculate diameter*
- Split node and rebalance tree
  - Merge clusters that are close to one anther
    - *Cluster-to-cluster distance; calculate CF of the merged cluster*

About BIRCH

- Single scan of data
  - CF tree is kept in memory
  - Size of the CF tree can be adjusted using the threshold value
- Cluster the leaf node clusters
  - More natural clusters
  - Sparse clusters detected as outliers
- Require the notion of centroid

Readings

- Textbook 7.5.1, 7.5.2