CS203 Programming with Data Structures

Sorting

Chengyu Sun
California State University, Los Angeles

Sorting

- Given an collection of elements, rearrange the elements so that they are in ascending or descending order
  - The *collection* is usually an array
  - The elements are comparable
- For example:

Before sorting:	30	10	15	21	18	25
After sorting:	10	15	18	21	25	30

Bubble Sort

Given an array of size `N`

For `i=0` to `N-1`
find the *smallest element in* the range `i+1` to `N-1`, and swap it with the element at position `i`
Done

- Or in other words
  - Find the smallest element and put it at 1st position
  - Find the second smallest element and put it ant 2nd position
  - …

Bubble Sort Example

	0	1	2	3	4	5
	30	10	15	21	18	25
After 1st iteration:	10	30	15	21	18	25

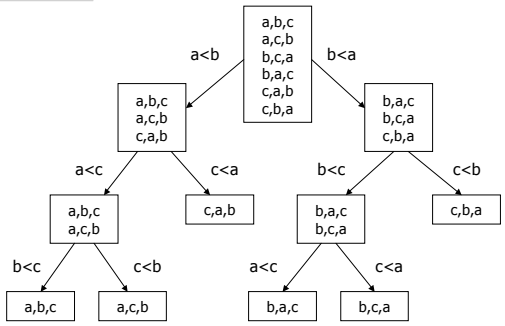
First iteration:

- i=0
- The smallest element in the rang 1 to 5 is 10
- Swap 10 and 30

Comparison and Swap

- Bubble sort
  - Number of comparisons??
  - Number of swaps??
- Can we improve on Bubble Sort??

Decision Tree



Properties of A Decision Tree

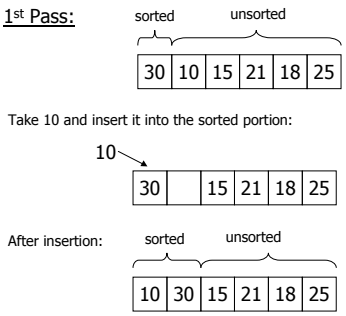
- Number of leaves: ??
- Height of the tree: ??

- *Any sorting algorithm that only uses comparisons between elements require at least O(NlogN) comparisons*

Insertion Sort

- Make `N-1` passes of the array
- At pass `p`,
  - the first `p` elements of the array are already sorted.
  - "insert" `a[p+1]` so the first `p+1` elements are sorted.

Insertion Sort Example



Insertion Sort Implementation

- Implementation #1:
  - Binary search for insertion position
  - shift elements to the right
  - Insert
- Complexity of pass P
  - logP comparisons
  - P/2 assignments

Insertion Sort Implementation

- Implementation #2:
  - Pair-wise swap
- Complexity of pass P
  - P/2 comparisons
  - P/2 swaps

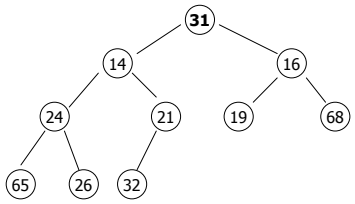
Insertion Sort Complexity

- Best case: ??
- Worst case: ??
- Average case: $O(N^2)$

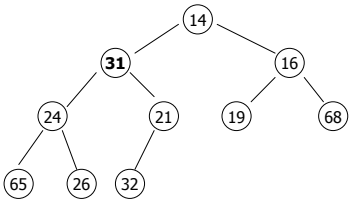
Heap Sort

- Heap sort strategy
  - Construct a heap with N *insertions*: O(??)
  - Construct a sorted array with N *removeMin*: O(??)
- *Can we construct the heap more efficiently (in linear time)??*
- *Can we perform heap sort without the extra space requirement??*

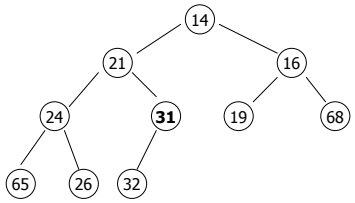
Percolate Down



Percolate Down



Percolate Down

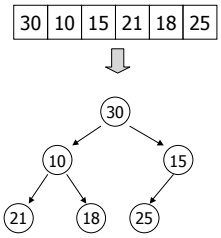


`percolateDown`

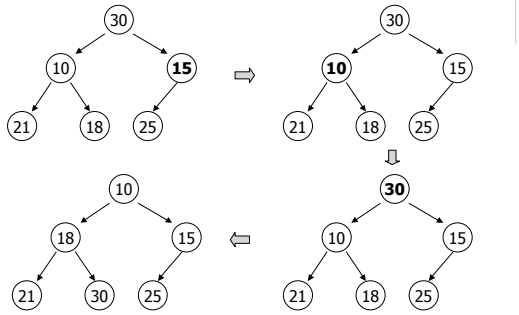
```
void percolateDown( int pos )
{
   int child;
   Comparable tmp = array[pos];

   while( pos*2 <= size )
   {
      child = pos * 2;
      if( child != size && array[child+1].compareTo(array[child]) < 0 ) child++;

      if( array[child].compareTo(tmp) < 0 ) array[pos] = array[child];
      else break;

      pos = child;
   }

   array[pos] = tmp;
}
```

Building A Heap

- Percolate down the non-leaf nodes *in reverse order*



Heap Building Example



Heap Building Complexity

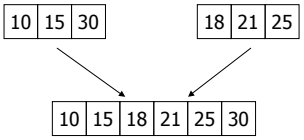
- The complexity of percolate down one node is: ??
- The complexity of percolate down all non-leaf nodes is: O(N)

Heap Sort Algorithm

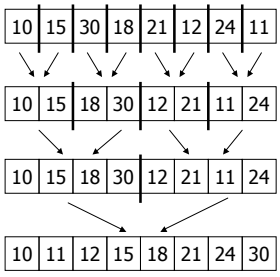
- Build a MaxHeap
- `removeMax()` then put the removed value into the last position

Merge Sort

- Merging two sorted arrays takes linear time



Merge Sort Example



Merge Sort Code …

```
Comparable tmpArray[];

void mergeSort( Comparable a[] )
{
   tmpArray = new Comparable[a.length];
   mergeSort( a, 0, a.length-1 );
}
```

… Merge Sort Code

```
void mergeSort( Comparable a[], int left, int right )
{
   if( left < right )
   {
      int mid = (left+right) / 2;
      mergeSort( a, left, mid );
      mergeSort( a, mid+1, right );
      merge( a, left, mid, right );
   }
}
```

About Merge Sort

- Complexity
  - T(1) = 1
  - T(N) = 2T(N/2) + N

  } O(NlogN)
- Rarely used in practice
  - Require extra space for the temporary array
  - Copying to and from the temporary array is costly

Quick Sort

- Fastest sorting algorithm *in practice*
- Complexity
  - Average case: O(NlogN)
  - Worst case: $O(N^2)$
- Easy to understand, very hard to code correctly

Quick Sort Algorithm

Given array A

1. If $|A|=1$ or 0, return
2. Pick any element v in A. v is called the pivot.
3. Partition $A-\{v\}$ (the remaining elements in A) into two disjoint groups A_1 and A_2
   - $A_1 = \{x \in A-\{v\} \mid x \leq v\}$
   - $A_2 = \{x \in A-\{v\} \mid x \geq v\}$
4. Return {quicksort(A_1), v, quicksort(A_2)}

Understand The Notations

A: | 30 | 10 | 15 | 21 | 18 | 25 |

v: 25

A_1: | 10 | 15 | 21 | 18 |

A_2: | 30 |

Observations About The Quick Sort Algorithm

- It should work
- It's not very clearly defined
  - How do we pick the pivot?
  - How do we do the partitioning?
  - How do we handle duplicate values?
- Why is it more efficient than Merge Sort?

Picking the Pivot

- Ideally, the pivot should leads to two equal-sized partitions
  - First element??
  - Random pick??
  - Median of (first, middle, last)

Partitioning …

A:

30	15	21	18	25	10

Pivot = median(30,21,10) = 21

1. Swap pivot to the last position

30	15	10	18	25	21

i j

… Partitioning

We want to move smaller elements to the left part of the array and larger elements to the right part, So:

2. Increase `i` until `a[i] > pivot`
   Decrease `j` until `a[j] < pivot`
   swap `a[i]` and `a[j]`

3. Repeat 2 until `i > j`

4. Swap `a[i]` and `pivot`

More Details

- Handing duplicates
- Small arrays
  - N>10: quick sort
  - N≤ 10: insertion sort

Exercise

- Implement `quickSort`